

Stock Assessment 101

Are you smarter than a 9th Grader?



NOAA
FISHERIES

Clay Porch
MREP Workshop
November 2023



Outline

Part 1

Stock assessment basics

Part 2

Surplus production and maximum sustainable yield

Part 3

Generating management advice from an assessment

What a stock assessment is not...

...interviewing people about what they caught...



or measuring fish

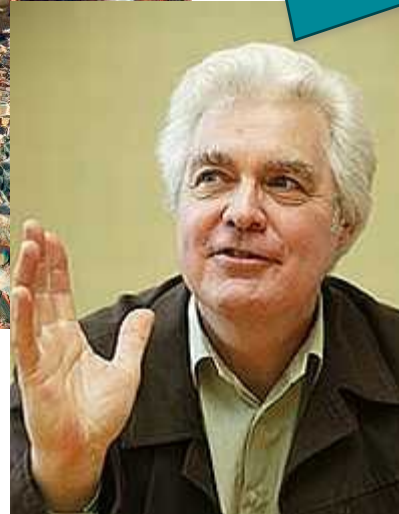


Photo Credit: Gary Moore, FWC

or counting fish...



"Fish are just like trees, except they are invisible and they keep moving around"



John Shepherd



Then just what is a stock assessment?

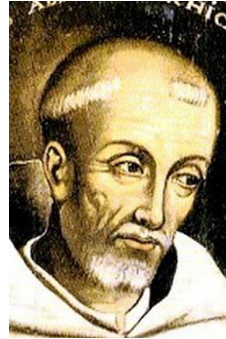
All the activities done to determine the condition of a stock and how it will respond to changes in fishing practices.

- Collecting data
- **Developing a model of the system**
- Using the model to make management recommendations



What is a model?

- A model is a simplified version of a real world system
- Models can never capture the full complexity of real systems
- The goal is to capture the general trends as accurately as possible

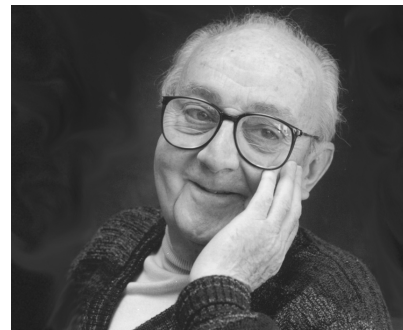


It is futile to do with more things that which can be done with fewer.

William of Ockham

All models are wrong, but some are useful.

... George E.P. Box



Remember those word problems from school?



A train leaves Madrid at 8:00 pm, traveling at 60 mph. Another train headed in the same direction leaves Madrid at 12:00 am, traveling at 90 mph.

How many hours after the second train leaves will it overtake the first train?

$$60(4 + T) = 90T$$

$$240 + 60T = 90T$$

$$240 = 30T$$

$$T = 8$$

Remember those word problems from school?



A train leaves Madrid at 8:00 pm, traveling at 60 mph. Another train headed in the same direction leaves Madrid at 12:00 am, traveling at 90 mph.

How many hours after the second train leaves will it overtake the first train?

This works well because we only want to get to the nearest hour, but what if we wanted to get to the nearest minute? Nearest second?

Stock assessments are big word problems

All stock assessments are based on the idea that if an **action** is taken on a population (e.g., catch), then there will be a **reaction**.

If we know an action was taken and can measure the reaction, then we will have learned something about the population.

If we learn enough, we can anticipate how the population will react to certain management actions.

But how can we learn much about the population based just on what comes back to the docks?

A simple example (with the math):

Catfish in a stocked pond



But how can we learn much about the population based just on what comes back to the docks?

Action: catch extracted from the population of fish

$$N_2 = N_1 - C_1$$

↑
Number of fish at beginning of year 2

↑
Number of fish at beginning of year 1

↙
Catch in year 1

But how can we learn much about the population based just on what comes back to the docks?

Action: catch extracted from the population of fish

Reaction: population decreases in abundance

$$N_2 = N_1 - C_1$$

$$U_1 = q N_1$$

Catchability coefficient

(fraction of population caught with one unit of effort)

Observed catch per unit effort at beginning of year 1

But how can we learn much about the population based just on what comes back to the docks?

Action: catch extracted from the population of fish

Reaction: population decreases in abundance

$$N_2 = N_1 - C_1$$

$$U_1 = q N_1$$

$$U_2 = q N_2$$

Lets do some math!



Observed catch per unit effort at beginning of year 2

A system of three equations and three unknowns


$$N_2 = N_1 - C_1$$

$$U_1 = q N_1$$

$$U_2 = q N_2$$

Goal: Express the unknown quantities q , N_1 and N_2 in terms of the known quantities C_1 , U_1 , and U_2

Step 1: Reduce the system of three equations and three unknowns to two equations and two unknowns

$$\begin{aligned} N_2 &= N_1 - C_1 \\ U_1 &= q N_1 \\ U_2 &= q N_2 \end{aligned} \quad \begin{aligned} U_1 &= q N_1 \\ U_2 &= q N_1 - q C_1 \end{aligned}$$


Step 2: reduce the system of two equations in two unknowns to a single equation

$$N_2 = N_1 - C_1$$

$$U_1 = q N_1$$

$$U_2 = U_1 - q C_1$$

$$U_1 = q N_1$$

$$U_2 = q N_1 - q C_1$$

$$U_2 = q N_2$$

$$= \frac{\quad}{\quad}$$

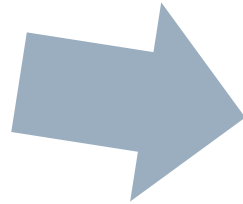


Step 3: Express unknown Ns in terms of knowns

$$N_2 = N_1 - C_1$$

$$U_1 = q N_1$$

$$U_2 = q N_2$$



$$U_1 = \frac{U_1 - U_2}{C_1} N_1$$

$$U_2 = \frac{U_1 - U_2}{C_1} N_2$$

$$q = \frac{U_1 - U_2}{C_1}$$

Substituting
for q

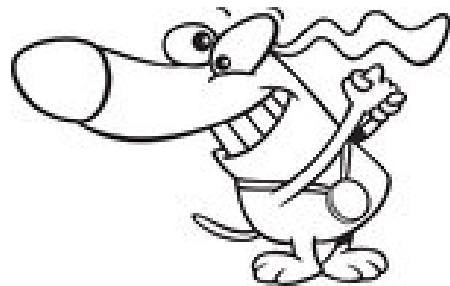
What is the
next step?

Step 3: Express unknown Ns in terms of knowns

$$U_1 = \frac{U_1 - U_2}{C_1} N_1$$
$$U_2 = \frac{U_1 - U_2}{C_1} N_2$$

Solving for N

$$N_1 = \frac{U_1 C_1}{U_1 - U_2}$$
$$N_2 = \frac{U_2 C_1}{U_1 - U_2}$$



or... $N_2 = N_1 - C_1$

An example

2000 fish are in the population at the start of year 1. During year 1, 1000 fish were caught. How many fish remain in the population at the beginning of year 2?

This was easy because we knew how many fish there were to begin with, but what about if we only saw what was coming in at the docks (catch and catch per unit effort)?

An example

$$C_1 = 1,000 \text{ fish}$$

$$U_1 = 1.0 \text{ fish per hour}$$

$$U_2 = 0.5 \text{ fish per hour}$$

$$N_1 = \frac{U_1 C_1}{U_1 - U_2} = \frac{1 * 1,000}{1 - 0.5} = 2,000$$



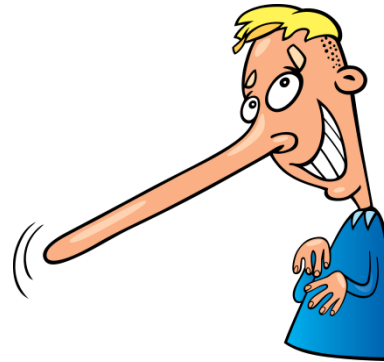
$$N_2 = N_1 - C_1 = 2,000 - 1,000 = 1,000$$

A complication: what if catch per unit effort is misreported

$$C_1 = 1,000 \text{ fish}$$

$$U_1 = 1.0$$

$$U_2 = \del{0.5} 0.2$$



$$N_1 = \frac{U_1 C_1}{U_1 - U_2} = \frac{1.0 * 1,000}{1.0 - 0.2} = 1,250$$

$$N_2 = N_1 - C_1 = 1,250 - 1,000 = 250$$

Another complication: what if catch per unit effort had increased despite the catch

$$C_1 = 1,000 \text{ fish}$$

$$U_1 = 1.0$$

$$U_2 = 2.0$$

$$N_1 = \frac{U_1 C_1}{U_1 - U_2} = \frac{1.0 * 1,000}{1.0 - 2.0} = -1,000$$

$$N_2 = N_1 - C_1 = -1,000 - 1,000 = -2,000$$

?

What is wrong with our assessment model?

Action: catch extracted from the population of fish

Reaction: population *increases* in abundance

$$N_2 = N_1 - C_1 + R$$

Recruitment of new fish into the population

Catch in year 1

Number of fish in year 1

Number of fish in year 2

The diagram shows the equation $N_2 = N_1 - C_1 + R$. A blue arrow points from the label 'Number of fish in year 2' to N_2 . Another blue arrow points from 'Number of fish in year 1' to N_1 . A third blue arrow points from 'Catch in year 1' to C_1 . A red arrow points from 'Recruitment of new fish into the population' to R .

What is wrong with our assessment model?

Action: catch extracted from the population of fish

Reaction: population *increases* in abundance

$$N_2 = N_1 - C_1 + R$$

$$U_1 = q N_1 \quad \text{catch per unit effort in year 1}$$

$$U_2 = q N_2 \quad \text{catch per unit effort in year 2}$$

Three equations, but four unknowns!

What to do?

$$N_2 = N_1 - C_1 + R$$

$$U_1 = q N_1 \quad \text{catch per unit effort in year 1}$$

$$U_2 = q N_2 \quad \text{catch per unit effort in year 2}$$

- Collect a new type of data: a survey to estimate R (did the owner restock?)
- Incorporate another year of catch per unit effort data

Adding another year of data

$$N_2 = N_1 - C_1 + R$$

$$N_3 = N_2 - C_2 + R$$

$$U_1 = q N_1$$

$$U_2 = q N_2$$

$$U_3 = q N_3$$

Five equations and five unknowns!

Solution starts getting more complicated!

$$q = \frac{2U_2 - U_1 - U_3}{C_2 - C_1}$$

$$N_1 = U_1(C_2 - C_1) / (2U_2 - U_1 - U_3)$$

$$N_2 = U_2(C_2 - C_1) / (2U_2 - U_1 - U_3)$$

$$N_3 = U_3(C_2 - C_1) / (2U_2 - U_1 - U_3)$$

$$R = N_1 - C_1 - C_2$$

Real stock assessments use more data and more complicated models



Age data



Size data



**Recreational
Monitoring**



**Scientific
Surveys**



**Biology and
Ecology**



**Commercial
Fisheries**

...and the math gets even harder!



$$N_{cah} = \sum_j \tilde{N}_{caj} T(h|j, a, s)$$

$$\tilde{N}_{c,a+1,h} = \begin{cases} R_c \tau_h & a = 0 \\ N_{cah} e^{-Z_{ash}} & 1 \leq a < A \\ 0 & a \geq A \end{cases} \quad Z_{asyh} = M_a + \sum_i F_{iasyh}$$

$$\mu\{R_c\} = \begin{cases} R_0 \phi_c \alpha^{1-\phi_c} \\ R_0 \frac{\alpha \phi_c}{1 + (\alpha - 1) \phi_c} \end{cases}$$

$$T(h|j, a, s) = \frac{\tau_{ash} e^{-x_{hj}/u_a}}{\sum_i \tau_{asi} e^{-x_{ij}/u_a}}$$

$$F_{iasyh} = q_{iy} v_{ia} f_{iy} \xi_{ia} \delta_{ish} / n\{s\}$$

$$\phi_{ck} = S_{ck} / S_{0k}$$

$$S_{ck} = \sum_a E_{ask} \sum_h N_{cahk}$$

$$S_{0k} = R_{0k} \sum_a E_{ask} \exp\left(-\sum_{j=1}^{a-1} M_{jk}\right)$$

$$-\log_e P(p_a | \Theta) = \sum_i \sum_y \sum_s n_{isy} \sum_a p_{iasy}^{obs} \log_e p_{iasy}$$

$$-\log_e P(p_l | \Theta) = \sum_i \sum_y \sum_s n_{isy} \sum_l p_{ilsy}^{obs} \log_e p_{ilsy}$$

$$C_{iasy} = \sum_h \frac{F_{iasyh}}{\xi_{ia} Z_{asyh}} \tilde{N}_{cah} (1 - e^{-Z_{asyh}})$$

$$C_{iy} = \sum_a C_{iaay}$$

$$I_{iy} = Q_i C_{iy} / f_{iy}$$

$$p_{iasy} = C_{iasy} / C_{iy}$$

$$p_{ilsy} = \sum_a p_{iaay} g\{l|a\}_{iy}$$

Inter-annual variations in f and q are modeled as first-order, lognormal auto-regressive processes, e.g.,

$$(8) \quad f_{iy} = \mu\{f_{iy}\} e^{\epsilon_{iy}} \\ \epsilon_{iy} = \rho\{f_{iy}\} \epsilon_{i,y-1} + \eta_{iy}$$

$$\log_e P(C|\Theta) = \begin{cases} 0.5 \sum_i \sum_y \sum_s \left(\frac{C_{iasy}^{obs} - C_{iasy}}{\sigma\{C_{iasy}\}} \right)^2 - \log_e (\sigma^2\{C_{iasy}\}) \\ 0.5 \sum_i \sum_y \sum_s \left(\frac{\log_e(C_{iasy}^{obs} / C_{iasy})}{\sigma\{\log_e C_{iasy}\}} \right)^2 - \log_e (\sigma^2\{\log_e C_{iasy}\}) \end{cases}$$

LOW

Data Needs

Model
Complexity

Output
Detail

HIGH

Methods

- Data Limited Models
 - CPUE + Catch (we just did this)
- Production Models
 - Lumped biomass (you will do this)
- Stage-based Models
 - Age or length
 - Many variations!
- Multi-species & Ecosystem



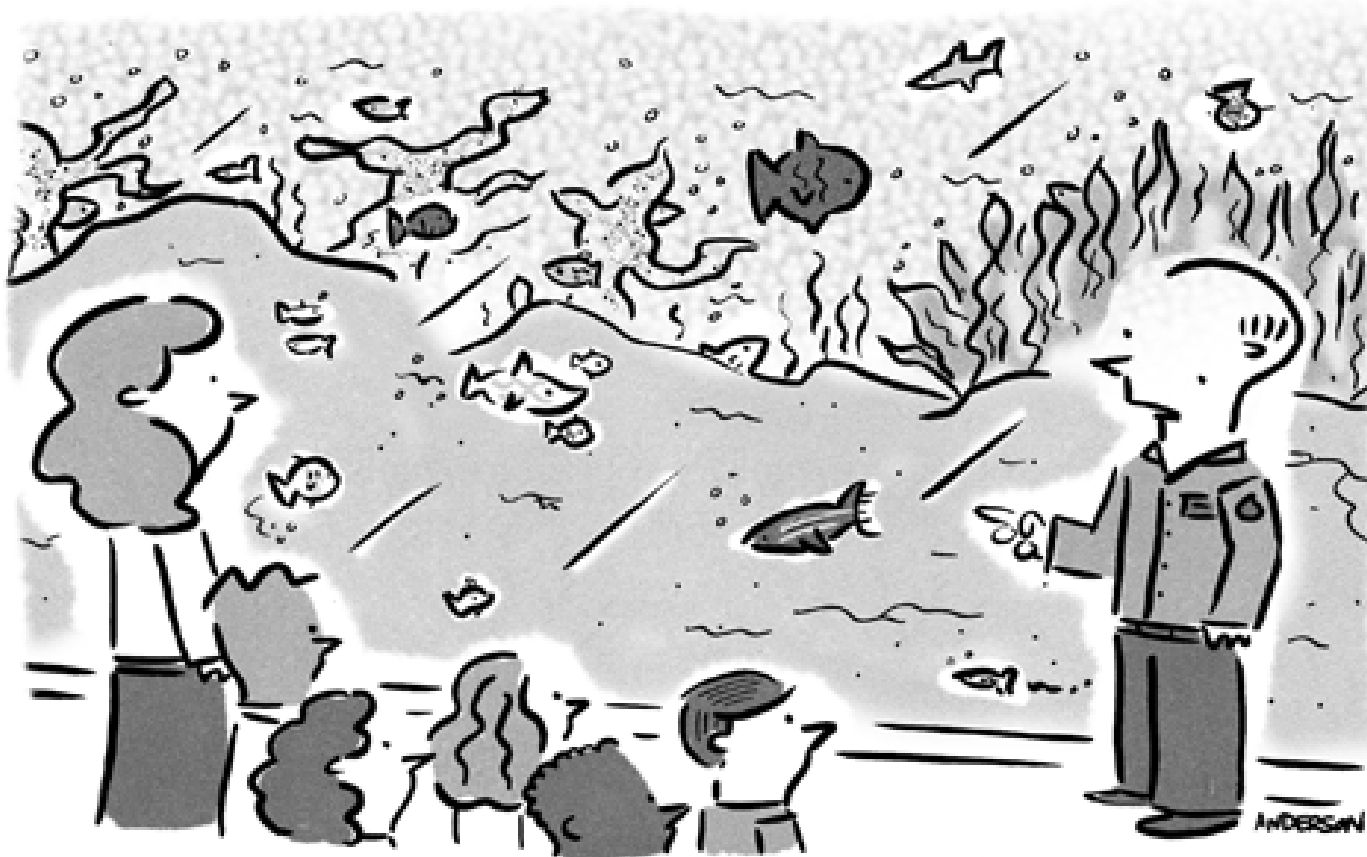
A practical definition of stock assessment

The science of developing a population model

- with reasonable ‘action’ equations (i.e., that capture the essential dynamics of the system)
- with parameters that can be estimated from the available ‘reaction’ observations (i.e., that can be fit to existing data)
- to provide advice on where the population is relative to management benchmarks and how it will respond to future management actions



Questions?



"No, that's Aquaman. I'm a stock assessment scientist."